Chapter 8

Complex Numbers I

Table 8-1: Complex conjugates illustrated

Operation	Note
$z(\bar{z}) = x^2 + y^2$	When conjugate complex numbers are multiplied together, the result is equal to the sum of the square of the real and the imaginary parts. The result is therefore always a non-negative real number.
$\overline{(z_1 z_2)} = \overline{z_1} * \overline{z_2}$	The product of two or more conjugates is equal to the product of their individual conjugates.
$z + \bar{z} = 2 * Re(z)$	The sum of a complex number and its conjugate is equal to its real part multiplied by 2.
$z - \bar{z} = 2j * Im(z)$	When a conjugate complex number is subtracted from its corresponding complex number, the result is the imaginary part multiplied by 2.

Table 8-2: Complex number forms

Expression	Name	Note
$z = x + jy$	rectangular or Cartesian	
$z = r(cos\theta \pm j\, sin\theta)$	polar	This is also called a trigonometrical form in some textbooks, while some authors consider it as a rectangular form.
$z = r\angle \pm \theta$	shortened polar	It is simply a representation, which can be considered as a derivative of the exponential form except that its angle is measured in degrees while the former is in radians.
$z = re^{\pm j\theta}$	exponential	

Table 8-3: Operations of complex numbers explained

Expression	Note
$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$	❑ Addition and subtraction carried out in rectangular form.
$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$	
$z_1 * z_2 = r_1 * r_2 \angle(\theta_1 + \theta_2)$	❑ Multiplication and division carried out in polar form. ❑ The same can be done in exponential form
$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\theta_1 - \theta_2)$	

Thank You