Chapter 18

Advanced Integration II

Table 18-1: Standard of integrals for powers of sine and cosine functions $n = 2$

$\boldsymbol{sin^2\, x}$	$\boldsymbol{cos^2\, x}$
$\int sin^2\, x\, dx = \int \left(\frac{1}{2} - \frac{1}{2} cos\, 2x\right) dx$ $= \boldsymbol{\frac{1}{2}x - \frac{1}{4} sin\, 2x + C}$	$\int cos^2\, x\, dx = \int \left(\frac{1}{2} + \frac{1}{2} cos\, 2x\right) dx$ $= \boldsymbol{\frac{1}{2}x + \frac{1}{4} sin\, 2x + C}$

Table 18-2: Standard of integrals for powers of sine and cosine functions $n = 3$

$\boldsymbol{\sin^3 x}$	$\boldsymbol{\cos^3 x}$
$\int \sin^3 x \,.\, dx = \int \sin x \,.\, \sin^2 x \,.\, dx$	$\int \cos^3 x \,.\, dx = \int \cos x \,.\, \cos^2 x \,.\, dx$
$= \int \sin x \,(1 - \cos^2 x).\, dx$	$= \int \cos x \,(1 - \sin^2 x).\, dx$
$= \int (\sin x - \sin x \,.\, \cos^2 x).\, dx$	$= \int (\cos x - \cos x \,.\, \sin^2 x).\, dx$
$= \int \sin x \,.\, dx - \int \sin x \,.\, \cos^2 x \,.\, dx$	$= \int \cos x \,.\, dx - \int \cos x \,.\, \sin^2 x \,.\, dx$
$= \int \sin x \,.\, dx + \int (-\sin x).\,(\cos x)^2.\, dx$	$= \int \cos x \,.\, dx - \int (\cos x).\,(\sin x)^2.\, dx$
$= -\cos x + \frac{1}{2+1}(\cos x)^{2+1} + C$	$= \sin x - \frac{1}{2+1}(\sin x)^{2+1} + C$
$\boldsymbol{= -\cos x + \frac{1}{3}\cos^3 x + C}$	$\boldsymbol{= \sin x - \frac{1}{3}\sin^3 x + C}$

Table 18-3: Standard of integrals for powers of sine and cosine functions $n = 4$

$\boldsymbol{\sin^4 x}$	$\boldsymbol{\cos^4 x}$
$\int \sin^4 x \, dx = \int (\sin^2 x)^2 . dx$	$\int \cos^4 x \, dx = \int (\cos^2 x)^2 . dx$
$= \int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)^2 . dx$	$= \int \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right)^2 . dx$
$= \int \left(\frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x\right) . dx$	$= \int \left(\frac{1}{4} + \frac{1}{2}\sin 2x + \frac{1}{4}\sin^2 2x\right) . dx$
$= \int \left(\frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\left[\frac{1}{2}\cos 4x + \frac{1}{2}\right]\right) . dx$	$= \int \left(\frac{1}{4} + \frac{1}{2}\sin 2x + \frac{1}{4}\left[\frac{1}{2} - \frac{1}{2}\sin 4x\right]\right) . dx$
$= \int \left(\frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x + \frac{1}{8}\right) . dx$	$= \int \left(\frac{1}{4} - \frac{1}{2}\sin 2x - \frac{1}{8}\sin 4x + \frac{1}{8}\right) . dx$
$= \int \left(\frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x\right) . dx$	$= \int \left(\frac{3}{8} - \frac{1}{2}\sin 2x - \frac{1}{8}\sin 4x\right) . dx$
$= \boldsymbol{\frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C}$	$= \boldsymbol{\frac{3}{8}x + \frac{1}{4}\cos 2x + \frac{1}{32}\cos 4x + C}$

Table 18-4: Standard of integrals for powers of sine and cosine functions $n = 5$

$\boldsymbol{\sin^5 x}$	$\boldsymbol{\cos^5 x}$
$\int \sin^5 x\,.dx = \int \sin^4 x\,.\sin x\,.dx$	$\int \cos^5 x\,.dx = \int \cos^4 x\,.\cos x\,.dx$
$= \int (\sin^2 x)^2.\sin x\,.dx$	$= \int (\cos^2 x)^2.\cos x\,.dx$
$= \int (1-\cos^2 x)^2.\sin x\,.dx$	$= \int (1-\sin^2 x)^2.\cos x\,.dx$
$= \int (1-2\cos^2 x+\cos^4 x).\sin x\,.dx$	$= \int (1-2\sin^2 x+\sin^4 x).\cos x\,.dx$
$= \int (\sin x-2\cos^2 x\,.\sin x+\cos^4 x\,.\sin x).dx$	$= \int (\cos x-2\sin^2 x\,.\cos x+\sin^4 x\,.\cos x).dx$
$= \int \sin x\,.dx-2\int \cos^2 x\,.\sin x+\int \cos^4 x\,.\sin x\,.dx$	$= \int \cos x\,.dx-2\int \sin^2 x\,.\cos x+\int \sin^4 x\,.\cos x\,.dx$
$= \int \sin x\,.dx+2\int -\sin x\,(\cos x)^2.dx-\int -\sin x\,(\cos x)^4.dx$	$= \int \cos x\,.dx-2\int \cos x\,.(\sin x)^2.dx+\int \cos x\,.(\sin x)^4.dx$
$= -\cos x+2\left[\frac{1}{3}\cos^3 x\right]-\left[\frac{1}{5}\cos^5 x\right]$	$= \sin x-2\left[\frac{1}{3}\sin^3 x\right]+\left[\frac{1}{5}\sin^5 x\right]$
$= -\boldsymbol{\cos x+\frac{2}{3}\cos^3 x-\frac{1}{5}\cos^5 x+C}$	$= \boldsymbol{\sin x-\frac{2}{3}\sin^3 x+\frac{1}{5}\sin^5 x+C}$

Thank You