Chapter 2

(2) Trigonometric Functions II

Table 2-1: Characteristics of sec function explained

$\mathbf{sec}\,\boldsymbol{x}$
1. As $sec\ x$ is a reciprocal of $\boldsymbol{cos\ x}$, it implies that it is undefined when $\boldsymbol{cos\ x = 0}$. Hence $\boldsymbol{sec\ x}$ has vertical asymptotes at these points, i.e., $\boldsymbol{x = \frac{1}{2}\pi \pm n\pi}$ where n is any positive integer.
2. As angle x approaches $\boldsymbol{\frac{1}{2}\pi}$ from the left, $\boldsymbol{cos\ x}$ is positive but approaches zero whilst $\boldsymbol{sec\ x}$ is positive but approaches infinity (or the vertical asymptote). On the other hand, if angle $\boldsymbol{x}$ approaches $\boldsymbol{\frac{1}{2}\pi}$ from the right, $\boldsymbol{cos\ x}$ is negative but approaches zero whilst $\boldsymbol{sec\ x}$ is also negative but approaches negative infinity.
3. $sec\ x$ is maximum when $\boldsymbol{cos\ x}$ is minimum and vice versa.
4. The maximum and minimum value for $\mathbf{sec\,x}$ are $\mathbf{-1}$ and $\mathbf{1}$ respectively. An exact reciprocal of $\boldsymbol{cos\ x}$.
5. Like $\boldsymbol{cos\ x}$, $\boldsymbol{sec\ x}$ has a periodicity of $\boldsymbol{2\pi}$ and $\boldsymbol{y}$ $-$**axis** as its line of symmetry.

Table 2-2: Characteristics of cosec function explained

$\mathbf{cosec}\,\boldsymbol{x}$
1. As $\boldsymbol{cosec\,x}$ is a reciprocal of $\boldsymbol{sin\,x}$, it implies that it is undefined when $\boldsymbol{sin\,x = 0}$. Hence $\boldsymbol{cosec\,x}$ has vertical asymptotes at these points, i.e., $\boldsymbol{x} = \pm\boldsymbol{n\pi}$ where n is any positive integer.
2. $cosec\ x$ is maximum when $\boldsymbol{sin\,x}$ is minimum and vice versa.
3. The maximum and minimum value for $\boldsymbol{cosec\,x}$ are $-\mathbf{1}$ and $\mathbf{1}$ respectively. An exact reciprocal of $\boldsymbol{sin\,x}$.
4. $\boldsymbol{cosec\,x}$ has a periodicity of $\mathbf{2}\boldsymbol{\pi}$ just like $\boldsymbol{sin\,x}$.
5. Since $\boldsymbol{sin\,x = cos\left(x - \frac{1}{2}\pi\right)}$, it follows that $\boldsymbol{cosec\,x = sec\left(x - \frac{1}{2}\pi\right)}$. This means that the graph of $\boldsymbol{cosec\,x}$ can be produced by translating $\boldsymbol{sec\,x\frac{1}{2}\pi}$ in the positive direction of $\boldsymbol{x}$ **−axis**.

Table 2-3: Characteristics of cot function explained

$\cot x$
1. As $\cot x$ is a reciprocal of $\tan x$, it means that it is undefined when $\tan x = 0$, which is the same as when $\sin x = 0$. Hence $\cot x$ has vertical asymptotes at these points, i.e., at $x = \pm n\pi$ where n is any positive integer.
2. $\cot x = 0$ when $\tan x$ is undefined, which implies that it crosses the x-axis at these points, i.e., $x = \frac{1}{2}\pi \pm n\pi$ where n is any positive integer. The reverse is the case, i.e. $\cot x$ is undefined, $\tan x$ crosses the x-axis
3. When $\tan x$ is positive and small, $\cot x$ will positive but large and vice versa. The same trend is observed for the negative y −axis region.
4. $\cot x$ has a periodicity of π just like $\tan x$.

Table 2-4: Arcsin function in the interval $-1 \leq \theta \leq 1$ and sine function in the interval $0 \leq \theta \leq 360°$ compared

$\sin x$	$\sin^{-1} x$
1. The domain is limited to $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. 2. The range is $-1 \leq \sin x \leq 1$. 3. The graph passes through the origin $(0, 0)$ only for the range taken. If we take other ranges, e.g., $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$, it will not go through the origin but will cross the x-axis. 4. The maximum and minimum points remain as 1 and -1 respectively. This is irrespective of the portion taken as interval, which fulfil the condition of one-to-one function.	1. The domain is $-1 \leq x \leq 1$. 2. The range is $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$. 3. The graph passes through the origin $(0, 0)$. 4. The coordinates of the endpoints are $\left(-1, -\frac{\pi}{2}\right)$ and $\left(1, \frac{\pi}{2}\right)$. This will remain unchanged even if there is a transformation in $\sin x$ as the domain must be limited to $-1 \leq x \leq 1$.

Table 2-5: Arccos function in the interval $-1 \leq \theta \leq 1$ and cosine function in the interval $0 \leq \theta \leq 360°$ compared

$\boldsymbol{cos\,x}$	$\boldsymbol{cos^{-1}\,x}$
1. The domain is limited to $\mathbf{0} \leq \boldsymbol{x} \leq \boldsymbol{\pi}$. 2. The range is $-\mathbf{1} \leq \boldsymbol{cos\,x} \leq \mathbf{1}$. 3. The graph passes through the x-axis at $\left(\frac{\pi}{2}, \mathbf{0}\right)$ and $\boldsymbol{y}$-axis at $(\mathbf{0}, \mathbf{1})$. If we take other ranges, e.g., $\boldsymbol{\pi} \leq \boldsymbol{x} \leq \mathbf{2}\boldsymbol{\pi}$, it will have different intercepts. 4. The maximum and minimum points remain as $\mathbf{1}$ and $-\mathbf{1}$ respectively. This is irrespective of the portion taken as interval, which fulfil the condition of one-to-one function.	1. The domain is $-\mathbf{1} \leq \mathbf{x} \leq \mathbf{1}$. 2. The range is $\mathbf{0} \leq \boldsymbol{cos^{-1}\,x} \leq \boldsymbol{\pi}$. 3. The graph passes through the $\mathbf{x}$-axis at $(\mathbf{1}, \mathbf{0})$ and $\mathbf{y}$-axis at $\left(\mathbf{0}, \frac{\pi}{2}\right)$. 4. The coordinates of the endpoints are $(-\mathbf{1}, -\boldsymbol{\pi})$ and $(\mathbf{1}, \boldsymbol{\pi})$. This will remain unchanged even if there is a transformation in $\boldsymbol{cos\,x}$ as the domain must be limited to $-\mathbf{1} \leq \mathbf{x} \leq 1$.

Table 2-6: Arctan function in the interval $-1 \leq \theta \leq 1$ and tangent function in the interval $0 \leq \theta \leq 360°$ compared

$\tan x$	$\tan^{-1} x$
1. The domain is limited to $-\frac{\pi}{2} < x < \frac{\pi}{2}$. 2. The range is $-\infty < \tan x < \infty$. Note that the range does not include the $\pm\infty$ as $\tan x$ is undefined at $\pm\pi$. We have vertical asymptotes at $x = \frac{\pi}{2}$ and $y = \frac{\pi}{2}$. 3. The graph passes through the origin $(0, 0)$ only for the range taken. If we take other ranges, e.g., $\frac{\pi}{2} < x < \frac{3\pi}{2}$, it will not go through the origin but will cross the x-axis.	1. The domain is $-\infty < x < \infty$. Note that the domain does not include the $\pm\infty$ as $\tan x$ is undefined at $\pm\pi$. We have vertical asymptotes at $y = -\frac{\pi}{2}$ and $y = \frac{\pi}{2}$. 2. The range is $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$. This is the interval of the answer you will get when you compute arctan function on a calculator for example 3. The graph passes through the origin $(0, 0)$.

Thank You