

Chapter 5

Solving Trigonometric Equations



Table 5-1: Changing the interval of transformed functions (vertical stretch) illustrated

	Equation	Original range	New range
1)	$y = \sin 2\theta$	$0 \leq \theta \leq 2\pi$	$0 \leq 2\theta \leq 4\pi$
2)	$y = \tan 3\theta$	$-180^\circ < \theta < 180^\circ$	$-540^\circ < 3\theta < 540^\circ$
3)	$y = \cos\left(\frac{3}{2}\theta\right)$	$0 \leq \theta < 180^\circ$	$0 \leq \frac{3}{2}\theta < 270^\circ$
4)	$y = \tan 0.5\theta$	$0 < \theta \leq 2\pi$	$0 < 0.5\theta \leq \pi$

Table 5-2: Changing the interval of transformed functions (horizontal translation) illustrated

	Equation	Original range	New range
1)	$y = \cos\left(\theta - \frac{\pi}{6}\right)$	$0 < \theta < \pi$	$-\frac{\pi}{6} < \left(\theta - \frac{\pi}{6}\right) < \frac{5}{6}\pi$
2)	$y = \sin(\theta + 35^\circ)$	$-180^\circ \leq \theta < 180^\circ$	$-145^\circ \leq (\theta + 35^\circ) < 215^\circ$
3)	$y = \tan(\theta - 0.5\pi)$	$0 < \theta \leq 4\pi$	$-\frac{1}{2}\pi < (\theta - 0.5\pi) \leq \frac{7\pi}{2}$

Table 5-3: Changing the interval of transformed functions (horizontal translation and stretch) illustrated

Equation	Original range	New range
1) $y = \sin\left(\frac{3}{5}\theta - 20^\circ\right)$	$-180^\circ \leq \theta \leq 360^\circ$	$-128^\circ \leq \left(\frac{3}{5}\theta - 20^\circ\right) \leq 196^\circ$
2) $y = \cos(2\theta + 60^\circ)$	$0^\circ < \theta < 180^\circ$	$60^\circ < (2\theta + 60^\circ) < 420^\circ$
3) $y = \tan\left(3\theta + \frac{1}{4}\pi\right)$	$\frac{1}{2}\pi \leq \theta < 2\pi$	$\frac{7}{4}\pi \leq \left(3\theta + \frac{1}{4}\pi\right) < \frac{25}{4}\pi$

Table 5-4: Addition Formulas

	Addition	Subtraction
Sine	$\sin(A + B) = \sin A \cos B + \sin B \cos A$	$\sin(A - B) = \sin A \cos B - \sin B \cos A$
Cosine	$\cos(A + B) = \cos A \cos B - \sin A \sin B$	$\cos(A - B) = \cos A \cos B + \sin A \sin B$
Tangent	$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$	$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Table 5-5: Double angle Formulas

Function	Identities
Sine	$\sin 2A = 2 \sin A \cos A$
Cosine	$\cos 2A = \cos^2 A - \sin^2 A$ $\cos 2A = 2\cos^2 A - 1$ $\cos 2A = 1 - 2\sin^2 A$
Tangent	$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

Table 5-6: Triple angle Formulas

Function	Identities
Sine	$\sin 3A = 3 \sin A - 4 \sin^3 A$
Cosine	$\cos 3A = 4 \cos^3 A - 3 \cos A$
Tangent	$\tan 3A = \frac{2 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

Table 5-7: Half angle Formulas

Function	Identities
Sine	$\sin A = 2 \sin \frac{1}{2} A \cos \frac{1}{2} A$
Cosine	$\cos A = \cos^2 \frac{1}{2} A - \sin^2 \frac{1}{2} A$ $\cos A = 2 \cos^2 \frac{1}{2} A - 1$ $\cos A = 1 - 2 \sin^2 \frac{1}{2} A$
Tangent	$\tan A = \frac{2 \tan \frac{1}{2} A}{1 - \tan^2 \frac{1}{2} A}$

Table 5-8: The factor formulas

Identities	
Sine	$\sin A + \sin B = 2\sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$ $\sin A - \sin B = 2\sin \frac{1}{2}(A - B) \cos \frac{1}{2}(A + B)$
Cosine	$\cos A + \cos B = 2\cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$ $\cos A - \cos B = -2\sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$

Table 5-9: The R formulas

Function	Sum	Difference
Sine	$a \sin x + b \cos x = R \sin(x + \alpha)$	$a \sin x - b \cos x = R \sin(x - \alpha)$
Cosine	$a \sin x + b \cos x = R \cos(x - \alpha)$	$a \sin x - b \cos x = R \cos(x + \alpha)$



Thank You

