Chapter 4

Trigonometric Identities II

Table 4-1: Factor formulas (sum to product) for trigonometric ratios

	Identities
Sine	$$\sin A + \sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$
	$$\sin A - \sin B = 2\sin\frac{1}{2}(A-B)\cos\frac{1}{2}(A+B)$$
Cosine	$$\cos A + \cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$
	$$\cos A - \cos B = -2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

Table 4-2: Factor (product-addition) formulas for trigonometric ratios

	Identities
Sine-Cosine	$\boxed{2\sin A\cos B = \sin(A+B) + \sin(A-B)}$ $\boxed{2\cos A\sin B = \sin(A+B) - \sin(A-B)}$
Cosine	$\boxed{2\cos A\cos B = \cos(A+B) + \cos(A-B)}$
Sine	$\boxed{2\sin A\sin B = -\cos(A-B) - \cos(A+B)}$

Table 4-3: The R addition formulas illustrated

Function	Sum	Difference
Sine	$\boldsymbol{a\ sin x + b\ cos x = R\ sin(x + \alpha)}$	$\boldsymbol{a\ sin x - b\ cos x = R\ sin(x - \alpha)}$
Cosine	$\boldsymbol{a\ sin x + b\ cos x = R\ cos(x - \alpha)}$	$\boldsymbol{a\ sin x - b\ cos x = R\ cos(x + \alpha)}$

Table 4-4: The R formulas - Author's method illustrated

Quadrant	Adjusted α (for Sine)	Adjusted α (for Cosine)
1st	α	$-\alpha$
2nd	$-\alpha$	$\alpha - 180°$
3rd	$\alpha - 180°$	$180° - \alpha$
4th	$180° - \alpha$	α

Table 4-5: Analysis for $R\,sin(x \pm \alpha)$

Expression	Quadrant	$\lvert\alpha\rvert$	Adjusted α	$R = \sqrt{a^2 + b^2}$	$R\,sin(x \pm \alpha)$
$\boldsymbol{sin\,x + cos\,x}$	First	$\left\lvert tan^{-1}\left(\frac{1}{1}\right)\right\rvert = 45°$	$45°$	$\sqrt{1^2 + 1^2} = \sqrt{2}$	$\sqrt{2}\,sin(x + 45°)$
$\boldsymbol{sin\,x - cos\,x}$	Second	$\left\lvert tan^{-1}\left(\frac{-1}{1}\right)\right\rvert = 45°$	$-45°$	$\sqrt{1^2 + (-1)^2} = \sqrt{2}$	$\sqrt{2}\,sin(x - 45°)$
$\boldsymbol{-sin\,x - cos\,x}$	Third	$\left\lvert tan^{-1}\left(\frac{-1}{-1}\right)\right\rvert = 45°$	$= (45° - 180°)$ $= -135°$	$\sqrt{(-1)^2 + (-1)^2}$ $= \sqrt{2}$	$\sqrt{2}\,sin(x - 135°)$
$\boldsymbol{-sin\,x + cos\,x}$	Fourth	$\left\lvert tan^{-1}\left(\frac{1}{-1}\right)\right\rvert = 45°$	$= (180° - 45°)$ $= 135$	$\sqrt{(-1)^2 + 1^2} = \sqrt{2}$	$\sqrt{2}\,sin(x + 135°)$

Table 4-6: Analysis for $R\,cos(x \pm \alpha)$

Expression	Quadrant	$\lvert\alpha\rvert$	Adjusted α	$R = \sqrt{a^2 + b^2}$	$R\,cos(x \pm \alpha)$
$\boldsymbol{sin\,x + cos\,x}$	1st	$\left\lvert tan^{-1}\left(\frac{1}{1}\right)\right\rvert = 45°$	$-45°$	$\sqrt{1^2 + 1^2} = \sqrt{2}$	$\sqrt{2}\,sin(x - 45°)$
$\boldsymbol{sin\,x - cos\,x}$	2nd	$\left\lvert tan^{-1}\left(\frac{1}{-1}\right)\right\rvert = 45°$	$= (45° - 180°)$ $= -135°$	$\sqrt{1^2 + (-1)^2} = \sqrt{2}$	$\sqrt{2}\,sin(x - 135°)$
$\boldsymbol{-sin\,x - cos\,x}$	3rd	$\left\lvert tan^{-1}\left(\frac{-1}{-1}\right)\right\rvert = 45°$	$= (180° - 45°)$ $= 135$	$\sqrt{(-1)^2 + (-1)^2}$ $= \sqrt{2}$	$\sqrt{2}\,sin(x + 135°)$
$\boldsymbol{-sin\,x + cos\,x}$	4th	$\left\lvert tan^{-1}\left(\frac{-1}{1}\right)\right\rvert = 45°$	$45°$	$\sqrt{(-1)^2 + 1^2} = \sqrt{2}$	$\sqrt{2}\,sin(x + 45°)$

Thank You