Chapter 12

Fundamentals of Differentiation

Table 12-1: The use of the limit sign illustrated

Example	Read as	Explanation
$\boldsymbol{x \to 2}$	x tends to 2	The value of the variable x approaches a numerical value of 2.
$\boldsymbol{x \to n}$	x tends to n	The value of the variable x approaches a variable n.
$\boldsymbol{n \to 0}$	n tends to 0	The value of the variable n approaches 0.

Table 12-2: The limit of $f(x)$ as x approaches 3 illustrated

(a)			(b)	
x	$y = f(x)$		x	$y = f(x)$
2.990	8.94428		3.010	9.06415
2.991	8.95026		3.009	9.05814
2.992	8.95623		3.008	9.05212
2.993	8.96221		3.007	9.04612
2.994	8.96819		3.006	9.04011
2.995	8.97417		3.005	9.03411
2.996	8.98016		3.004	9.0281
2.997	8.98614		3.003	9.0221
2.998	8.99213		3.002	9.01611
2.999	8.99812		3.001	9.01011
3.000	9.00412		3.000	9.00412

Table 12-3: Examples of limit of function

$f(x)$	Limit	Answer	Note
$\boldsymbol{x^n+1}$	$\lim_{n\to 0} f(x)$	2	This is because when n approaches 0, $\boldsymbol{x^n} \to x^0 \to 1$.
$\boldsymbol{x^n+1}$	$\lim_{n\to\infty} f(x)$	▪ $x^n, for\ x > 1$ ▪ $1, for\ x < 1$ ▪ $2\ for\ x = 1$	▪ This is because when n approaches ∞, $\boldsymbol{x^n}$ becomes the dominant term. ▪ This is because when n approaches ∞, $\boldsymbol{x^n}$ becomes very small. ▪ This is because $1^\infty = 1$.
$\boldsymbol{\left(1+\frac{1}{x}\right)^x}$	$\lim_{x\to\infty} f(x)$	$e \approx 2.718281 \ldots$	We will be using this soon.
$\boldsymbol{\sin\theta}$	$\lim_{x\to 0} f(x)$	$\boldsymbol{\theta}$	Provided the angle is: i) Very small, and ii) It is measured in radian. We will be using this soon.
$\boldsymbol{\frac{\sin\theta}{\theta}}$	$\lim_{x\to 0} f(x)$	1	Provided the angle is measured in radian. We will be using this soon.

Table 12-4: Gradient of a non-linear function using numerical method illustrated

x_1	y_1	x_2	y_2	Δx	Δy	m
0.5	1.25	0.550	1.30	0.05	0.05	1.05
0.5	1.25	0.548	1.30	0.05	0.05	1.05
0.5	1.25	0.546	1.30	0.05	0.05	1.05
0.5	1.25	0.544	1.30	0.04	0.05	1.04
0.5	1.25	0.542	1.29	0.04	0.04	1.04
0.5	1.25	0.540	1.29	0.04	0.04	1.04
0.5	1.25	0.538	1.29	0.04	0.04	1.04
0.5	1.25	0.536	1.29	0.04	0.04	1.04
0.5	1.25	0.534	1.29	0.03	0.04	1.03
0.5	1.25	0.532	1.28	0.03	0.03	1.03
0.5	1.25	0.530	1.28	0.03	0.03	1.03
0.5	1.25	0.528	1.28	0.03	0.03	1.03
0.5	1.25	0.526	1.28	0.03	0.03	1.03
0.5	1.25	0.508	1.26	0.01	0.01	1.01
0.5	1.25	0.506	1.26	0.01	0.01	1.01

x_1	y_1	x_2	y_2	Δx	Δy	m
0.5	1.25	0.524	1.27	0.02	0.02	1.02
0.5	1.25	0.522	1.27	0.02	0.02	1.02
0.5	1.25	0.520	1.27	0.02	0.02	1.02
0.5	1.25	0.518	1.27	0.02	0.02	1.02
0.5	1.25	0.516	1.27	0.02	0.02	1.02
0.5	1.25	0.514	1.26	0.01	0.01	1.01
0.5	1.25	0.512	1.26	0.01	0.01	1.01
0.5	1.25	0.510	1.26	0.01	0.01	1.01
0.5	1.25	0.508	1.26	0.01	0.01	1.01
0.5	1.25	0.506	1.26	0.01	0.01	1.01

Table 12-5: Notations for a derivative of a function illustrated

Notation	Read as
$\boldsymbol{\frac{dy}{dx}}$	dee y, dee x (or dee y by dee x or dee y over dee x)
$\boldsymbol{f'(x)}$	f-prime of x (or f-prime x)
$\boldsymbol{y'}$	y-prime
$\boldsymbol{\frac{d}{dx}(fx)}$	dee dee x of f function of x
$\boldsymbol{\dot{y}}$	y dot

Table 12-6: Derivatives of inverse trigonometric functions

Function	$\frac{dy}{dx}$	Function	$\frac{dy}{dx}$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}, x \neq \pm 1$	$\sec^{-1} x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}, x \neq \pm 1{,}0$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}},$ $x \neq \pm 1$	$\mathrm{cosec}^{-1} x$	$\frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$ $x \neq \pm 1{,}0$
$\tan^{-1} x$	$\frac{1}{1+x^2}$	$\cot^{-1} x$	$\frac{-1}{1+x^2}$

Table 12-7: Notations for higher derivatives illustrated

Degree	Notation	Read as	Alternative notations
1st derivative	$\frac{dy}{dx}$	dee y dee x	$f'(x); \dot{y}\ ; y'\ ; \frac{d}{dx}(y)$
2nd derivative	$\frac{d^2y}{dx^2}$	dee two y dee x squared	$f''(x); \ddot{y}\ ; y''; \frac{d}{dx}\left(\frac{dy}{dx}\right)$
3rd derivative	$\frac{d^3y}{dx^3}$	Dee three y dee x cubed	$f'''(x); \dddot{y}\ ; y'''; \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$
4th derivative	$\frac{d^4y}{dx^4}$	Dee four y dee x quadrupled	$f'^{v}(x); \frac{d}{dx}\left(\frac{d^3y}{dx^3}\right)$
nth derivative	$\frac{d^ny}{dx^n}$	Dee n y dee x n	$\frac{d}{dx}\left(\frac{d^{n-1}y}{dx^{n-1}}\right)$

Thank You

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